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t-dependence of pion production in $\pi^- p \to \pi^0\pi^0 n$

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Abstract. As the E852 and the GAMS Collaboration published their data for the dependence of the reaction $\pi^- p \to \pi^0\pi^0 n$ on the momentum transfered to the nucleon, questions rose, whether the $K\bar{K}$-molecule picture might be ruled out by their experiments. Here we report on our analysis of these data and our investigations into concerns raised in discussions, like the role of the $NN\pi\pi\pi$ contact term. We find the molecule picture capable to explain the experimental data.

PACS. 11.55.Fv Dispersion relations – 11.80.Gw Multichannel scattering – 13.75.Gx Pion-baryon interactions – 13.75.Lb Meson-meson interactions

1 Introduction

Both the GAMS Collaboration [1] and the BNL E852 experiment [2] observed a peculiar behaviour of the $f_0(980)$-resonance when measuring the t-dependence of the reaction $\pi^- p \to \pi^0\pi^0 n$. While for low momentum transfer t to the nucleon, *e.g.* $-t < 0.1\,\mathrm{GeV}^2$, the resonance shows as a distinct dip, a sharp peak is seen at the same position for large t, *e.g.* $-t > 0.2\,\mathrm{GeV}^2$. Based on this observation claims were made that there must be a hard $q\bar{q}$-component in the $f_0(980)$-resonance [3]. Looking at this phenomenon within the framework of the Jülich meson exchange model for $\pi\pi$ scattering [4] we also investigate claims that the $K\bar{K}$-molecule, generating the $f_0(980)$-resonance within this model, only forms because the independent empirical t-channel form factors can be used to generate the needed attraction in the $K\bar{K}$-channel. Instead of using those arbitrary form factors we develop a self-consistent procedure to calculate the meson form factors microscopically in the s-channel and use a dispersion relation to generate the t-channel form factors. In sect. 2 we describe the self-consistent procedure used to calculate these form factors. Section 3 gives a short description of how pion production is handled in our model. As we do not consider pion production by the $NN\pi\pi\pi$ contact term we give a short discussion on the influence of this term in sect. 4. Our results for the t-dependence observed in pion production are presented in sect. 5.

2 Self-consistent form factors

Since within the Jülich meson exchange model s- and t-channel form factors were treated independently, the possibility existed that the $K\bar{K}$-molecule was an artefact of too point-like t-channel form factors, which overestimate the generated attraction. In the old model the bare s-channel vertex f^0 gets dressed by the non-pole part of the T-matrix (T^{NP}) and the s-channel resonances aquire a self-energy contribution by means of the Bethe-Salpeter equation, which both do not happen in the t-channel. To account for those mechanisms also in the t-channel we calculate the s-channel form factor ($\Gamma(s)$) as an empirical form factor F plus dressing by meson exchanges (1), (2) and use dispersion relation (3) to continue the form factor to the t-channel. Here, F is chosen to be a formal interpolation of the old s- and t-channel form factors:

$$\Gamma(s) f^0(s) = f(s) = (1 + T^{\mathrm{NP}} G)\tilde{f}^0 \,, \tag{1}$$

$$\tilde{f}^0 = F f^0 = \frac{4\Lambda^4 + M^4}{4\Lambda^4 + 4\omega^4(k)} f^0 \,, \tag{2}$$

$$\Gamma(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\Im(\Gamma(t'))}{t' - t} + \text{pole contributions}\,. \tag{3}$$

The same idea is applied to the self-energy. Care has to be taken in both cases since the empirical form factor introduces poles, which have to be accounted for in the dispersion relations. As the formulas for both form factors and self-energy depend on the latter two to be known, we start from the empirical values of the old model and use them to calculate a first estimate, which then is the starting point to calculate a second estimate and so forth until we obtain a self-consistent solution. During this iteration we make sure that $\pi\pi$ phase shifts and inelasticities keep their good agreement with experiment. The differences between the old and new form factors are not too large so that this calculation justifies the empirical form factors used in the model before.

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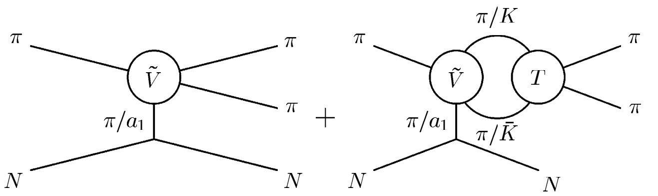

Fig. 1. Pion production as Born amplitude plus final-state interaction. For the potential $\tilde{V}$ only the π/a_1 emitted by the nucleon is allowed to be off its mass shell.

One aspect regarding the change in the s-channel form factors is worth mentioning. The poles introduced by the empirical form factor get closer to the physical region as we switch to our interpolated form factor. This is necessary since we want to use a dispersion relation in one variable, which means, that the poles of the old form factor lie in the region we want to continue to. This is why the poles of the new form factor had to contain a non-vanishing imaginary part. These poles, even though their contribution decreases during iteration, stay part of the self-consistent form factors. As those poles originate from an empirical form factor, which is as its predecessors acausal, we might shortly discuss their influence on our calculation even though we have not yet introduced our model for pion production. There are two main aspects in which the poles show up: Firstly they cause a pronounced decay into $K\bar{K}$. Secondly they cause a great part of the excess strength in the pion production at low invariant two pion masses in the region of high momentum transfer t to the nucleon (fig. 3b) below).

3 Pion production

In order to compare to the data provided by E852 and GAMS, we apply the model depicted in fig. 1. That is, we use a Born amplitude together with subsequent final-state interaction. We take the Jülich meson exchange description together with the self-consistent form factors described in sect. 2 to model the final-state interaction, *i.e.* the final-state interaction is calculated in a three-dimensional reduction of the Bethe-Salpeter equation with a kernel as described in [4] plus the four-pion contact terms added in [5]. As the production vertex $\tilde{V}$ is concerned, we only allowed the leg connected to the nucleon to be off-shell. $\tilde{V}$ contains t-channel exchanges for ρ and K^*, as well as s-channel resonances and the four-pion contact terms. The π and the a_1 emitted at the nucleon vertex have usually high momentum. So the question rises whether the Blankenbeclar-Sugar propagator we are using is a good enough approximation. To check this, we replace them by Regge propagators with the parameters taken from [6]. This replacement had only slight influence on the result. So we are justified in showing the results for the Blankenbeclar-Sugar propagators only. To be consistent with the experimental analysis we further adopted the use of exponential form factors at the nucleon vertex fitting their parameters to the t-dependence of the total cross-section.

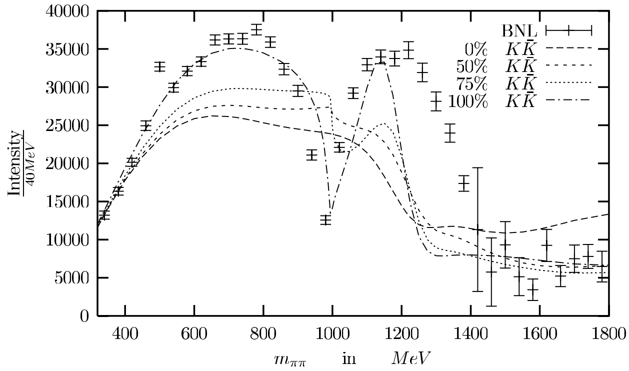

Fig. 2. Development of $f_0(980)$ while the coupling to the $K\bar{K}$-channel is increased. The data shown is taken from the BNL E852 experiment [2].

4 NN$\pi\pi\pi$ contact term in the production

At low momentum transfer to the nucleon and low invariant pion masses a sizeable contribution to the production comes from diagrams starting with the emission of a pion from the nucleon followed by a four-pion contact interaction, which at those masses is not yet cut off by its form factor. So there is the chance that this class of diagrams gets cancelled by diagrams starting with a $NN\pi\pi\pi$ point vertex. In the case of pion deuteron scattering this effect has for example been discussed by [7]. Two points should be noted in this respect:

1. When the ρ was introduced as a gauge particle our Lagrangian acquired a slightly different form compared to [7] plus an additional $\pi\pi\pi\pi$ contact term.
2. The cancellation occurs only at threshold in the chiral limit of vanishing pion mass. This can be seen when looking at the production amplitudes shown in the appendix of [8] (diagrams A) and B)).

Looking at the production in the discussed momentum transfer region one sees only partial cancellation of the amplitudes corresponding to the ungauged Lagrangian. Since these give only minor contribution to the overall production, an effect on the final result can hardly be noticed. This should be even more the case, if the $NN\pi\pi\pi$ contact term gets its own appropriate form factor instead of sharing the form factor for the $\pi\pi\pi\pi$ vertex as we chose to ensure maximum cancellation.

5 Results

We find that the formation of a $K\bar{K}$-molecule is by no means an artefact of the unconstrained t-channel form factors. This is demonstrated in fig. 2, where the low momentum transfer pion production is shown as calculated for self-consistent form factors. To demonstrate that the origin of the $f_0(980)$ seen here as a sharp dip really lies within the $K\bar{K}$-channel we start from the case of an uncoupled $\pi\pi$-channel (long dashes), which shows no distinct features. As we increase the coupling to the $K\bar{K}$-channel

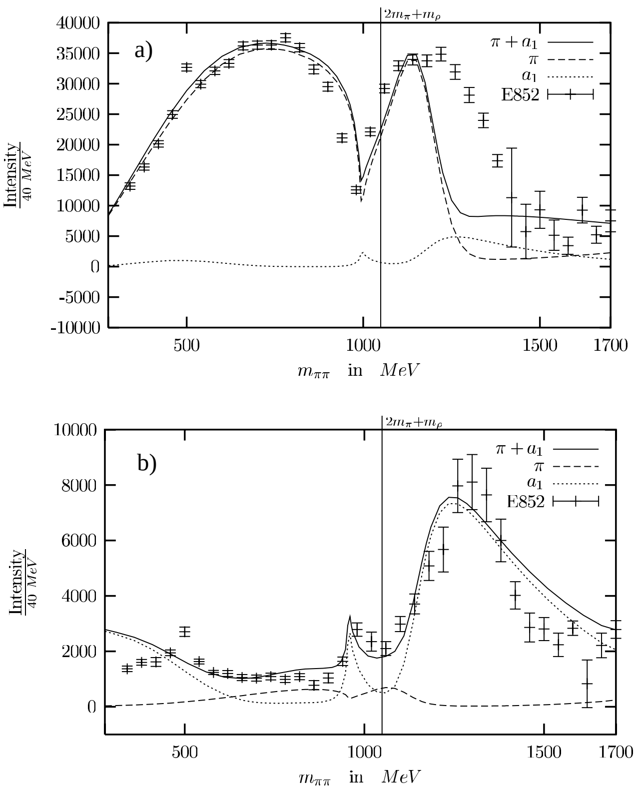

Fig. 3. t-dependence of pion production in the S-wave. a) shows low momentum transfer, *i.e.* $-t = 0.03\,\mathrm{GeV}^2$ calculations with data from [2] averaged for $0.01 < -t < 0.1\,\mathrm{GeV}^2$ and b) shows high momentum transfer, *i.e.* $-t = 0.8\,\mathrm{GeV}^2$ calculations with data from [2] averaged for $0.4 < -t < 1.5\,\mathrm{GeV}^2$.

as well as the interaction in this channel, a cusp structure (dashes) shows, which evolves into the sharp dip in the fully coupled case (dash-dotted line). This shows how the $f_0(980)$ is formed as a $K\bar{K}$-molecule. When looking at the full model (dash-dotted line in fig. 2) one clearly sees that we miss a lot of strength above 1.2 GeV. We will investigate this missing strength together with the t-dependence of the data. When looking at fig. 3 we see, apart from the afore-mentioned one, two further features that we miss. One is the strength at low invariant pion masses, which we already traced back to the close poles of our empirical s-channel form factors. The other is the distinct peak at 500 MeV, which we should not reproduce, since it stems from kaon decays and has nothing to do with the reaction under investigation.

But now let us discuss the t-dependence before analysing the missing strength. Looking at fig. 3 one sees the experimental t-dependence nicely reproduced. This is due to a change in the production mechanism. While in the low-t case depicted in 3a) the production via pion emission is dominant (dashed line), the production in the high-t case is mainly due to the a_1. This is demonstrated by the dotted line in 3b). The peak emerging in 3b) instead of the dip in 3a) is generated by destructive interference of the first diagram in fig. 1 and the parts of the second diagram having gone through the $K\bar{K}$ channel. When looking at the missing strength at high invariant two pion masses it should be stressed, that there are further channels to be considered and that there might be some problems with the S-wave data around 1.2–1.4 GeV as strong G-waves appear, which could be a misinterpretation of the S-wave strength [9]. Nevertheless, we tried to fill up the second bump of 3a) just by resonances. Using just a broader version of the $f_0(1370)$ fails to reproduce the high momentum transfer structure in 3b), whereas there is no problem to fill the missing strength by an additional resonance. This shows, that the t-dependence gives valuable information on the resonance structure. We also introduced a genuine, *i.e.* $q\bar{q}$, $f_0(980)$ in our model, which by itself was not able to reproduce the measured inelasticities, but cannot be excluded to contribute as an admixture to the $f_0(980)$-resonance if the coupling is small.

6 Summary

Good agreement with experiment has been achieved using our model for self-consistent form factors even though we had to move the unphysical singularities closer to the physical region, which has caused some artefacts like an increase of production close to threshold. The good agreement with experiment covers scattering phases and inelasticities as well as the t-dependence of the S-wave amplitudes. Further we had no problem to generate the attraction in the $K\bar{K}$-channel needed to form a molecule even though our form factors are now heavily constrained. Comparing our new form factors with the old empirical ones, we found good agreement and would consider the use of empirical form factors in the Jülich meson exchange model to be justified. To investigate the energy region up to 2 GeV, the $\rho\rho$, $\eta\eta$, $\pi\eta$ channels have to be considered [10].

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